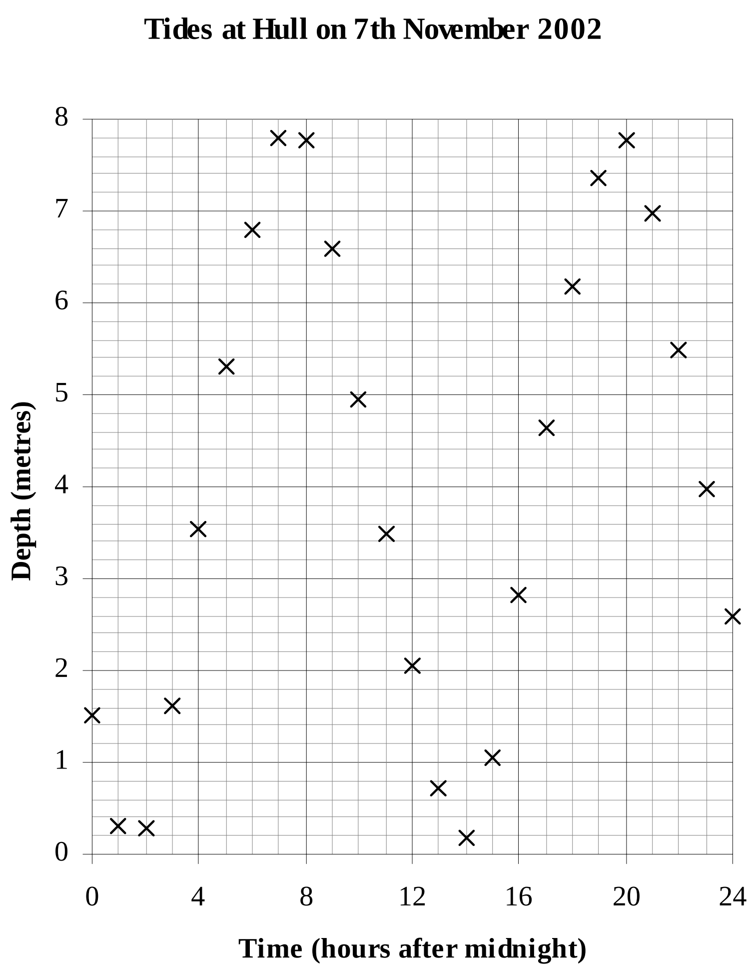
Tides

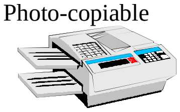
Data Sheet

The gravitational effect of the sun and moon causes the depth of sea water at coastal locations to vary with time. This variation is very important to navigators of all types of ships and boats as well as to those planning coastal engineering or construction projects or water sporting activities. The times of high and low tides are published for many coastal locations in the UK and in some cases more detailed data is given.

The table and graph below show how the water depth varied at a location in Hull over the course of twenty-four hours on 7th November 2002.

Time	Depth (metres)
00:00	1.52
01:00	0.32
02:00	0.27
03:00	1.61
04:00	3.55
05:00	5.31
06:00	6.79
07:00	7.80
08:00	7.77
09:00	6.58
10:00	4.96
11:00	3.48
12:00	2.06
13:00	0.71
14:00	0.19
15:00	1.05
16:00	2.83
17:00	4.64
18:00	6.19
19:00	7.37
20:00	7.76
21:00	6.97
22:00	5.48
23:00	3.98
00:00	2.59





Tides

Assignment

Your assignment is to find and evaluate functions to model the data given on the Data Sheet.

Which of the following types of functions do you think would be suitable to model all or part of the data given in the table and graph? Briefly explain the reasons for your choice of functions.

Linear? Quadratic? Power? Exponential? Trigonometric?

Choose **two** different types of function to model different parts of the data set
either:

- one type of function for the full data set, and another for part(s) of the data set,

or:

- two different types of function for different sections of the data set.

Explain how you chose the parameters of your functions and how your functions are related to basic functions of their type.

Plot graphs to compare the given data with values given by your models.
(At least one of your graphs should be drawn using a graphic calculator or computer software.)

Consider how errors or inaccuracies in the given data may affect your models and explain how, in general terms, the functions you found could be different.

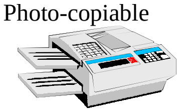
This table gives the times of the high and low tides at Hull on 7th November 2002 and the corresponding depths of water. Compare these with the times and depths predicted by your model(s).

Low Tide	Time	01:31	13:54
	Depth (m)	0.10	0.18
High Tide	Time	07:29	19:55
	Depth (m)	7.94	7.77

Each ship or boat requires a minimum depth of water.
Choose one of the following depths: 1 m, 2 m, 3 m, 4 m, 5 m, 6 m, 7 m.
Use the graphs of your functions to predict the times at which the depth of water is greater than or equal to the particular value you have chosen. Use your functions to check your answers.
Compare your predictions with the actual times given below:

Depth	≥ 1 metre	≥ 2 metres	≥ 3 metres	≥ 4 metres	≥ 5 metres	≥ 6 metres	≥ 7 metres
Times	before 00:22 02:38 to 12:45 after 14:58	03:12 to 12:02 after 15:34	03:43 to 11:21 16:06 to 23:42	04:15 to 10:38 16:38 to 22:59	04:49 to 09:58 17:13 to 22:18	05:27 to 09:22 17:52 to 21:40	06:10 to 08:43 18:38 to 20:59

Summarise your findings and consider the effectiveness of each of your functions as a model of the data. Indicate clearly when your functions can be considered valid models for the data and describe any limitations they have.



Teacher Notes

Unit Advanced Level, Working with algebraic and graphical techniques

Notes
This assignment is intended to address many parts of the first coursework portfolio requirement given below:

1a
• find two different types of function to model different parts of the same data set (*either:*
  • *one function for the full data set, another for part of this*, or
  • *two different functions for different sections of the data*.)
where you:
• plot at least one set of data and one function using a graphic calculator or computer software
• consider the effectiveness of each function as a model
• use your graphs of functions to predict what will happen in cases for which you have no data
• explain how the functions you used are related to the basic functions of their type
• consider qualitatively how errors or inaccuracies in your data may affect your model of the situation by considering how, in general terms, the function you found could be different
	• choose **appropriate** functions to model different parts of your data set
	• explain how you chose the parameters of your functions referring to how they relate to the basic function of their type
	• indicate clearly the circumstances in which your functions can be considered valid models for your data

b use key features of graphs including each of (i) intercepts with axes, (ii) gradients, (iii) changes and trends in gradients, (iv) local maximum and minimum points, for functions that model real situations in order to solve problems and explain how the function relates to the real situation	• indicate clearly the key features of models and solutions to problems on your graphs and describe these clearly in real world terms in your report
c use algebraic techniques to solve problems for (i) a polynomial model and (ii) one other model from the following types: trigonometric, exponential or logarithmic.	• show clearly the stages of your working when using algebra • use correct algebraic notation

In this assignment students should produce work that covers all of 1a and some of 1b and 1c, (depending on which functions and methods they choose to use).

The assignment is written in a way that encourages students to work independently and this will enable more able students to achieve high marks for their portfolio. However some students will find it difficult to make decisions about which functions to use and how to approach the tasks. You could use discussion with individuals or the whole class to help them decide which of the types of functions listed are the most suitable before they try to find particular functions for themselves. Weaker students can be given guidance at any point, but of course this should be reflected in their final mark.

There is a variety of different functions that can be used to model the data and a range of methods for completing some of the tasks set. Some suggestions and possible solutions are given below. The data is provided on an Excel spreadsheet, and this has been used to produce these solutions, but your students could use a graphic calculator and/or do some work by hand if preferred.

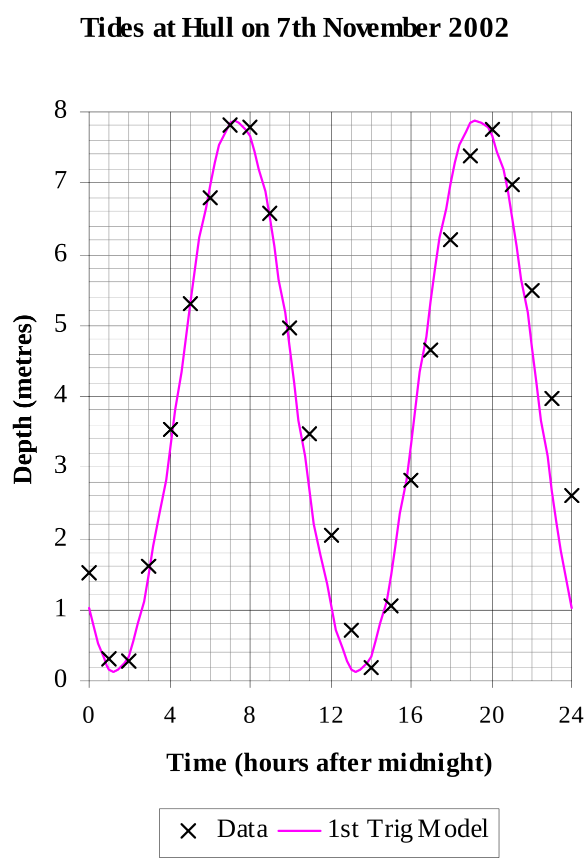
Suggestions and Possible Solutions

Trigonometric models can be used to model the full data set.
Two of the many possible functions are shown here.

This graph shows the original data and the function:
$d = 3.9\sin(30t - 130)^\circ + 4$

The parameters of this function were found by estimating the amplitude, period, central value and phase shift from the original data and graph.

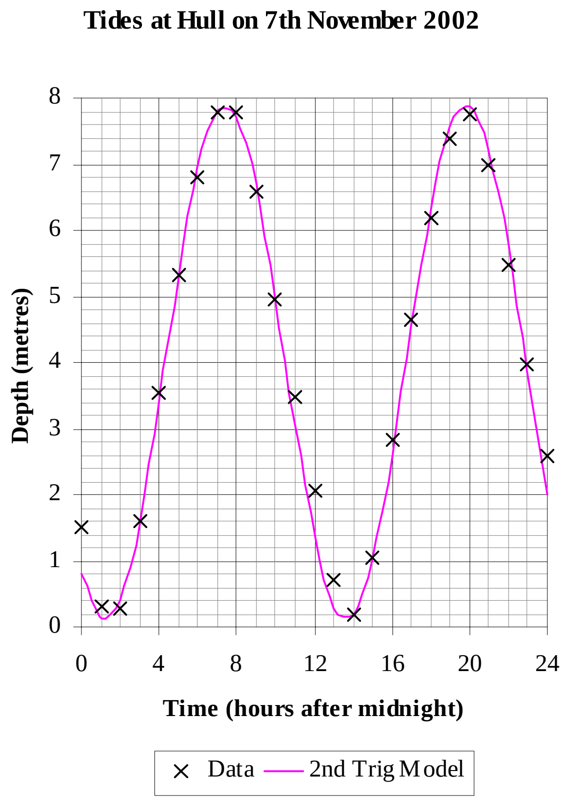
Although a reasonable model for the first 10 hours or so, the graph shows that this function deviates more from the data points at later times.



The second trigonometric function (shown here) is :
$d = 3.9\sin(29t - 125)° + 4$

In this case trial and improvement was used to find parameters that give a reasonable fit to more of the original data.

Many other sine and cosine functions provide possible models.



Linear Functions can be used to model some parts of the data set.

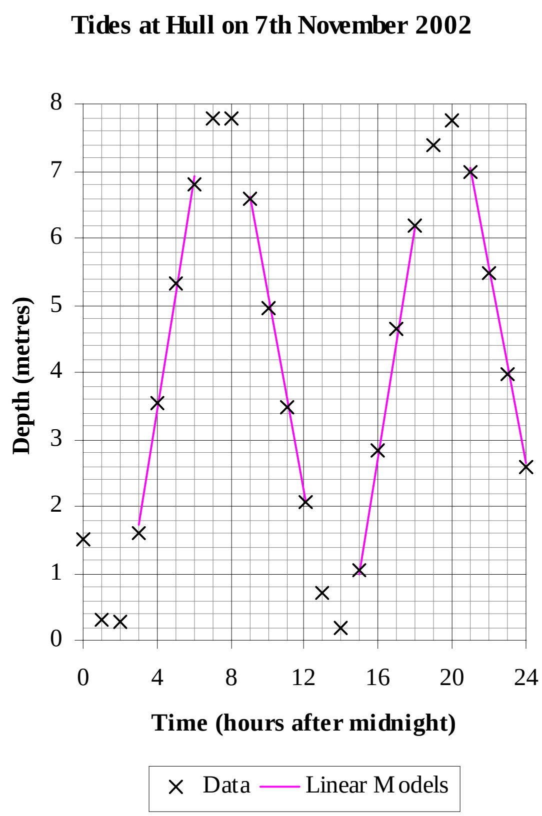
A graphic calculator was used to give the following functions for the lines of best fit shown here.

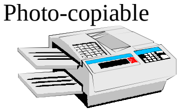
$3 \le t \le 6$	$d = 1.73t - 3.47$
$9 \le t \le 12$	$d = -1.50t + 20.1$
$15 \le t \le 18$	$d = 1.72t - 24.8$
$21 \le t \le 24$	$d = -1.46t + 37.7$

The Excel spreadsheet could also have been used to give linear trendlines.

A linear function for each section can also be found by substituting the co-ordinates from two of the data points into $d = mt + c$ then solving simultaneous equations to find m and c. This has the advantage of providing students with the opportunity of working with algebra.

Yet another method would be to draw lines of best fit by eye, then find gradients and intercepts.





Quadratic Functions can be used to model other parts of the data set.

A graphic calculator was used to give the following quadratic functions for the curves shown here.

$0 \le t \le 3$
$d = 0.635t^2 - 1.88t + 1.53$

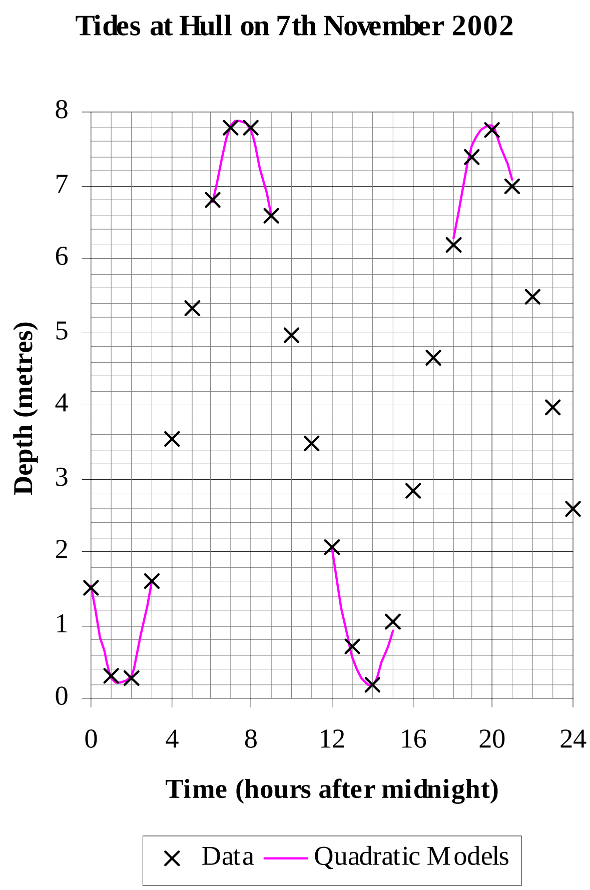
$6 \le t \le 9$
$d = -0.550t^2 + 8.18t - 22.5$

$12 \le t \le 15$
$d = 0.553t^2 - 15.3t + 106$

$18 \le t \le 21$
$d = -0.493t^2 + 19.5t - 185$

The Excel spreadsheet could also have been used to give quadratic trendlines.

Substituting the co-ordinates from three data points into $d = at^2 + bt + c$ then solving simultaneous equations for *a*, *b* and *c* would also give a quadratic function for each section whilst having the advantage of providing students with another opportunity of working with algebra.



The functions students suggest may be good or poor models of the data. Even poor models have some value if students are able to evaluate them by comparing them with the original data and reach an appropriate conclusion.

In finding, drawing and evaluating functions to model the data, students should be able to cover all of requirement 1a for their coursework portfolios.

In the assignment students are also asked to use their functions to predict the times and depths of water at high and low tides and also the times at which the depth of the water is less than or equal to particular values. This gives students more opportunities to produce some work that can contribute to requirements 1b and 1c.